Energy supply systems: state-of-the-art and challenges

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Challenges of the energy transition:

- Target for renewable electric power in 2030: 50%
- Power generation from renewable energies is volatile
- Need for fast reaction to cover load differences → gas-to-power using gas turbines
Overview

1. Mathematical Modeling
2. Solution procedure
3. Numerical results
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A coupled power-gas network

- Network is modeled as a directed graph
- Edges $\triangleq$ different components (pipelines, compressors, \ldots )
  $\rightarrow$ algebraic equations, ODEs/PDEs
- Nodes $\triangleq$ Boundary and coupling conditions
  $\rightarrow$ Boundary conditions - e.g. external demand profile
  $\rightarrow$ Coupling conditions (Kirchhoff laws, mass conservation)
• **AC powerflow equations** (with parameters $G_{kj}$, $B_{kj}$):

\[
P_k = \sum_{j \in \text{node}} |V_k||V_j| \left[ G_{kj} \cos(\phi_k - \phi_j) + B_{kj} \sin(\phi_k - \phi_j) \right] \\
Q_k = \sum_{j \in \text{node}} |V_k||V_j| \left[ G_{kj} \sin(\phi_k - \phi_j) - B_{kj} \cos(\phi_k - \phi_j) \right]
\]

• **PQ-/loads, PV-/generators, V\phi/slack bus**
Gas model

- Dynamics in every pipe: **isentropic Euler equations**
  - Mass flow: \( \partial_t \rho + \partial_x q = 0 \)
  - Momentum balance: \( \partial_t q + \partial_x (p(\rho) + q^2/\rho) = -\lambda(q/\rho)\frac{q|q|}{2D\rho} \)
  - Pressure law: \( p(\rho) = \kappa \rho^\gamma \) (with \( 1 \leq \gamma \leq 3 \))
Conservation of mass and equality of pressure as coupling conditions:

\[ p(\rho_1(t)) = p(\rho_2(t)) = p(\rho_3(t)) \]

\[ q_1(t) = q_2(t) + q_3(t) \]
Compressor stations are installed to compensate the loss of pressure:

- Change of pressure: \( p_{\text{out}}(t) = p_{\text{in}}(t) + u(t) \)
- Conservation of mass: \( q_{\text{out}}(t) = q_{\text{in}}(t) \)
- Energy consumption: \( P(p_{\text{in}}(t), q_{\text{in}}(t), p_{\text{out}}(t), q_{\text{out}}(t)) \)
Gas consumption depends on the electric power $P(t)$:

- Gas consumption$^4$: \[ e(P(t)) = a_0 + a_1 P(t) + a_2 (P(t))^2 \]
- Mass conservation: \[ q_{\text{out}}(t) = q_{\text{in}}(t) + e(P(t)) \]
- Pressure equality: \[ p(\rho_{\text{out}}(t)) = p(\rho_{\text{in}}(t)) \]

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Coupling gas-to-power

\[ q_{\text{out}}(t) = q_{\text{in}}(t) + e(P(t)) \]

- Conditions for the well-posedness of the model:\(^5\):
  1. \(2p'(\rho) + \rho p''(\rho) \geq 0\) and \(6p'(\rho) + 6\rho p''(\rho) + \rho^2 p'''(\rho) \geq 0\)
  2. \(p \to \infty \text{ für } \rho \to \infty\) or
     \(p'(\rho) \leq \frac{p(\infty) - p(\rho)}{\rho}\) for all \(\rho > 0\),
  3. There exists a \(\eta \in [0, 1]\) such that \(\rho^{2\eta} p'(\rho) \xrightarrow{\rho \to 0} s_0 \in \mathbb{R}^+\) or
     \(\lim_{\rho \to 0} p'(\rho) = 0\) and \(2p'(\rho) - \rho p''(\rho) \geq 0\) for all \(\rho > 0\).
  4. \(0 \leq e(P(t)) < e_{\text{max}}(\rho_{\text{in}}, q_{\text{in}}, \rho_{\text{out}}, q_{\text{out}})\)

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\(^6\) Herty, Müller, Sikstel, *Coupling of compressible Euler equations*, Vietnam J. Math., 2019
Coupling gas-to-power

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  4. \( 0 \leq e(P(t)) < e_{\text{max}}(\rho_{\text{in}}, q_{\text{in}}, \rho_{\text{out}}, q_{\text{out}}) \)

- Similar results also exist for the full Euler equations\(^6\).

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\(^6\) Herty, Müller, Sikstel, *Coupling of compressible Euler equations*, Vietnam J. Math., 2019
Optimal control problem

\[
\begin{align*}
\min_{u(t)} & \quad \int_{0}^{T} P(p_{\text{in}}(t), q_{\text{in}}(t), p_{\text{out}}(t), q_{\text{out}}(t)) \, dt \\
\text{s.t.} & \quad \text{power flow equations,} \\
& \quad \text{isentropic Euler equations,} \\
& \quad \text{equations for compressor station}^{7} \text{ and gas turbine,} \\
& \quad \text{initial and boundary conditions (e.g. load profiles),} \\
& \quad \text{state constraints (e.g. bounds on pressure)}
\end{align*}
\]

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Optimal control problem

- Optimization task: \( \min_u J(y(u), u) \)

\( u.d.N. \quad g_{\text{min}} \leq g(y(u), u) \leq g_{\text{max}} \)

\( u_{\text{min}} \leq u \leq u_{\text{max}} \)

- State \( y(u) \) is determined by the control \( u \) via state equation

\( E(y, u) \overset{!}{=} 0 \)

- Apply interior point solver IPOPT

- Computation of derivatives using **adjoint equations**:
  - Adjoint equation: \( \left( \frac{\partial}{\partial y} E(y, u) \right)^T \xi = - \left( \frac{\partial}{\partial y} f(y, u) \right)^T \)
  - Computation of derivatives: \( \frac{d}{du} f(y(u), u) = \frac{\partial}{\partial u} f(y, u) + \xi^T \frac{\partial}{\partial u} E(y, u) \)
Disretization

- Application of ANACONDA toolbox\(^7\)

- **Time discretization**: \(\Delta t = 15\) minutes, \(t_j = j\Delta t\) \((j \in \{0, \ldots, 48\})\)

- **Space discretization** (isentropic Euler equations): \(\Delta x_e \approx 1\) km

- Implicit box scheme\(^8\) for \(y_t + f(y)_x = g(y)\):

\[
\frac{Y_{j-1}^{n+1} + Y_j^{n+1}}{2} = \frac{Y_{j-1}^{n} + Y_j^{n}}{2} - \frac{\Delta t}{\Delta x} \left( f(Y_j^{n+1}) - f(Y_{j-1}^{n+1}) \right) + \frac{\Delta t}{2} \left( g(Y_{j-1}^{n+1}) + g(Y_j^{n+1}) \right)
\]

where \(Y_j^n \approx y(x, t)\) for \(x \in [(j - \frac{1}{2})\Delta x, (j + \frac{1}{2})\Delta x]\), \(t \in [n\Delta t, (n + 1)\Delta t]\)

\(^7\) Kolb, *Simulation and Optimization of Gas and Water Supply Networks*, Verlag Dr. Hut, 2011

\(^8\) Kolb, Lang, Bales, *An implicit box scheme for subsonic compressible flow with dissipative source term*, Numer. Algorithms, 2010
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Gas network is a snapshot of the GasLib-40-network\textsuperscript{9}

Electrical grid is „case 9“ from MATPOWER\textsuperscript{10}

Time horizon $T = 12$ hours, $\Delta t = 15$ minutes, $\Delta x_e \approx 1$ km

Lower bound on pressure is 41 bar at node G7

First we do a simulation with inactive compressor station, then cost minimization of compressor station

\textsuperscript{9}Humpola et al., \textit{GasLib – A Library of Gas Network Instances}, ZIB Report, 2015

Test case

![Diagram of energy supply system]

- **G1** to **G2**: Compressor station
- **G4** to **G5**: Gas turbine
- **G6** to **G7**: Further components

**Graphical Data:***

- **Power [p.u.]:**
  - **P at P5**
  - **Q at P5**
  - Time [hours]: 0, 5, 10
  - Power values: 0, 1.5, 2

- **Flow [m³/s]:**
  - **Inflow at G1**
  - Time [hours]: 0, 5, 10
  - Flow values: 0, 110, 140

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Test case

- Evolution of pressure at node G7:

![Graph showing pressure evolution at node G7 over time](image)
Test case

- Evolution of pressure at node G7:

![Graph showing the evolution of pressure at node G7](image)

- The graph illustrates the flow of pressure through nodes G1 to G7, with G7 being the target node for analysis.

- The dot-dashed line represents the pressure at G7, while the gray line indicates the lower pressure bound at G7.

- The timeline on the x-axis represents time in hours, ranging from 0 to 12, and the y-axis represents pressure in bar, ranging from 40 to 44.
Test case

- Evolution of pressure at node G7:

![Diagram showing the evolution of pressure at node G7 with nodes and arrows indicating the flow of pressure.](image)

![Graph showing the pressure at G7 over time with labels for pressure at G7, lower pressure bound, and optimized pressure bound.](graph)
Summary and future work

Summary:

- A mathematical setup for a coupled gas-to-power network is introduced.
- The discretization scheme even works efficiently for large-scale instances.
- Optimal control framework can be computed quite efficiently.

Future work:

- Feedback stabilization for the coupled network problem.
- Stochastic power demand and/or chance constraints.
- Use gas network as temporary storage.